tecnotest®

Le prove geotecniche alla luce delle nuove normative

Seminario

14 marzo 2008

Gentili signori,

desidero ringraziarvi per aver accettato di partecipare a questo nostro seminario dedicato alle prove sulle terre, ma soprattutto sento il dovere di fare delle presentazioni che mi paiono d' obbligo.

Come avrete saputo, da settembre c'è stato un avvicendamento nella proprietà di Tecnotest, ed io, assieme ai dottori Bosi ed Onofri, ho intrapreso questa nuova ed interessante avventura nel campo delle apparecchiature per prove sui materiali da costruzione. Come ho avuto occasione di dire in questi mesi, pur provenendo noi da settori diversi, ci ha accomunati l' interesse per l'ottima organizzazione, per la filosofia di qualità e per i commenti positivi che Tecnotest ha raccolto ovunque. Insomma un orologio prezioso e preciso destinato a segnare le ore per tantissimo tempo.

A proposito di tempo, quest' anno la nostra azienda festeggia i primi 40 anni di attività, e abbiamo così pensato di festeggiarla aggiungendo un' ulteriore gamma di preziosità a quanto essa offre attualmente. Infatti, e' recentissima la sigla di un accordo di cooperazione con l' americana GCTS, leader nelle macchine per prove cicliche, e di conseguente distribuzione dei suoi prodotti sul nostro mercato. Non a caso, ho detto cooperazione perché, a differenza di altri, la nostra intenzione non è quella di allungare la lista dei prodotti offerti, ma di dare pieno supporto prima durante e dopo la vendita di questi complessi apparecchi che spaziano dalle prove sulle terre a quelle sugli asfalti.

Per fare questo, seguendo la filosofia Tecnotest, ci siamo detti che prima dobbiamo conoscere e capire per poi essere in grado di consigliare e quindi assistere. Per far ciò abbiamo stretto un rapporto di collaborazione con l'Università di Bologna, dipartimento strade ed aeroporti, ed un ingegnere da loro selezionato, ma alle nostre dipendenze, sarà a Phoenix, sede di GCTS, per un periodo di diversi mesi per poi diventare, verso fine anno, il project leader di questa divisione.

Per ora, ancora una volta, il nostro prezioso dott. Mambrini, a cui passo la parola, vi informerà sulle nostre attuali problematiche e poi sulla prima macchina ciclica che abbiamo pensato di presentarvi sul mercato, la colonna risonante.

Grazie e buon lavoro a tutti.

Nuovi indirizzi normativi orientati alle prove cicliche

M. Mambrini
R&S Tecnotest - Modena

Nel 1981, fu imposto alle Regioni di fissare proprie Norme per prevenire il rischio sismico segnatamente agli strumenti urbanistici (Legge 741).

Compatibilmente con i tempi ed i modi propri della burocrazia, iniziano a vedersi gli sviluppi locali di quella legge.

Per fare l'esempio dell'Emilia-Romagna, da circa un anno i tecnici chiamati a redigere i piani urbanistici dispongono di *Linee guida* per l'approccio alle problematiche sismiche ai fini della microzonazione del territorio.

Si tratta di un corposo dossier con una serie d'istruzioni differenziate per i vari gradi d'approfondimento che articolano la ricerca.

Per le zone classificate 1, 2 e 3, laddove le analisi di livello intermedio non escludano liquefazione, densificazione, instabilità e magnificazione, nonché per le opere di rilevante interesse pubblico, la procedura risulta particolarmente complessa, attingendo ad argomenti propri dell'ingegneria geotecnica sismica.

La caratterizzazione dei terreni riguarda sia l'ambito statico sia quello dinamico: oltre ai metodi intesi alla misura in situ di V_s (velocità delle onde di taglio), sono richieste prove di laboratorio per determinare le leggi di variazione del modulo di taglio G e dello smorzamento D relativamente all'ampiezza della deformazione di taglio γ.

Per richiamare qualche nozione in proposito, in un ciclo d'isteresi disegnato nel piano τ/γ, (sforzo/deformazione),

$G = \tau_{pp} / \gamma_{pp}$

$D = W_D/(4\pi W_s)$

dove τ_{pp} e γ_{pp} sono le relative misure tra picco e picco, W_D e W_s sono gli integrali campiti in figura 1.

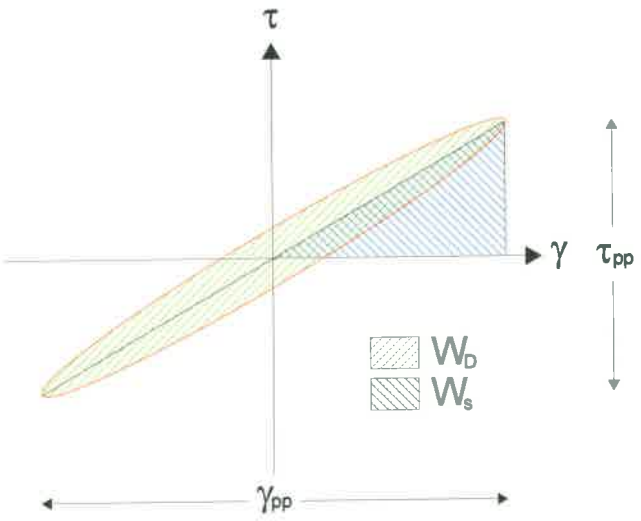

fig. 1

La relazione generale che lega G e V_s è la seguente:

$G = \rho V_s^2$

con ρ = densità del materiale.

Tali parametri entrano a vario titolo nei calcoli di previsione degli effetti locali attesi per un input sismico di caratteristiche statisticamente riferibili all'area.

Per la determinazione delle leggi tenso-deformative e degli smorzamenti viscosi, le suddette Linee guida fanno espresso riferimento a *prove di colonna risonante* e di *taglio torsionale ciclico*.

In Italia, questi sono tests sperimentali eseguiti da qualche istituto di ricerca e, dunque, fuori del circolo dei laboratori normalmente a disposizione dei Progettisti, mentre nei Paesi dove la prevenzione dagli effetti dei terremoti è di più antica data, il patrimonio delle macchine cicliche è assai più ricco.

Anche nella progettazione geotecnica e strutturale c'è aria di novità: la definitiva ed univoca cogenza delle "Norme tecniche per le costruzioni" richiederà una sorta di rivoluzione culturale per i tecnici addetti ai lavori.

Ingegneri e Geologi dovranno calarsi in metodiche che lasciano poco spazio alla superficialità e all'improvvisazione.

Parimenti, a responsabilità meglio definite occorrerà contrapporre dati sperimentali più circostanziati e, infine, la tecnologia dovrà conciliare la riduzione dei costi con l'incremento della sicurezza.

La recente normativa, sposando l'approccio prestazionale ai diversi stati limite possibili durante la vita utile dell'opera, ampia lo spettro dei parametri geotecnici d'interesse a quelli relativi alle piccole deformazioni, che non riguardano solo l'ambito dinamico, ma anche quello statico, giacché in quest'ultimo lavorano di regola il terreno e la struttura nelle condizioni d'esercizio.

Tali livelli di deformazione, dove peraltro il comportamento del suolo non è lineare, sono difficilmente riproducibili con le attrezzature di laboratorio tradizionali.

L'uso di queste ultime porta spesso a sovrastimare i rapporti tenso-deformativi, con conseguente aggravio economico delle opere geotecniche e strutturali.

Più adatta allo scopo è, appunto, l'apparecchiatura di taglio torsionale e colonna risonante.

E' dunque prevedibile che proprio tale strumento rappresenti il passaporto d'ingresso alle prove cicliche per il laboratorio di rango.

Ma di che cosa si tratta?

In pratica, una sorta di cella triassiale alloggia un provino cilindrico con la base appoggiata ad un plinto fisso e la testa connessa ad un dispositivo elettromagnetico che imprime alla medesima determinate sollecitazioni torsionali cicliche o monotoniche (vedi figura 2).

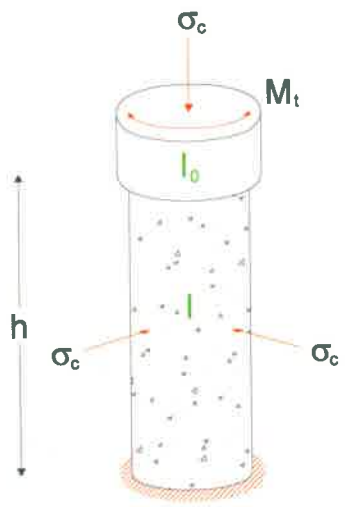

fig. 2

Generalmente la prova si conduce in condizioni non-drenate, dopo aver saturato e consolidato isotropicamente il campione con procedure tradizionali.

La differenza basilare tra la prova di taglio ciclico e di colonna risonante sta nella velocità di deformazione: se questa è sufficientemente lenta da poter considerare trascurabile il contributo inerziale di provino e cappello-motore si attua la prima, altrimenti si attua la seconda.

L'approccio fisico è assai differente nei due casi, giacché nel taglio si misurano in via diretta le risposte del materiale, com'è prassi nell'ambito statico, mentre nella colonna risonante queste sono da determinarsi in via indiretta.

Ciò è imposto dalle forze d'inerzia prodotte dalle elevate accelerazioni che investono il complesso provino-cappello-motore; i singoli contributi sono da calcolarsi tramite un'analisi dell'equilibrio dinamico del sistema quando questo risuona alla sua frequenza naturale.

La dimostrazione fisica che ciò sia possibile richiede qualche equazione differenziale (si veda a tal proposito la memoria dell'amico Manuel Padilla, artefice della GCTS), ma quella complessità non ci deve assillare più di tanto, visto che a noi interessano le formule risolutrici.

Dunque, la prova di colonna risonante comporta innanzi tutto che si attui il fenomeno della risonanza.

La frequenza di risonanza corrisponde a quella che produce la massima deformazione di taglio quando l'oscillatore forzante sollecita il provino con ampiezza e frequenza costanti.

Storicamente, le apparecchiature dedicate a tale tipo di test erano composte da una serie di sottosistemi da sincronizzarsi manualmente: oltre alla cella coi relativi elettromagneti, sensori, condotti, ecc., serviva un oscillatore variabile, un amplificatore di potenza, un analizzatore di segnali, un oscilloscopio e così via, strumenti che richiedevano all'operatore competenze e manualità in uno spettro teorico-applicativo estremamente ampio e disarmante; da qui il timore reverenziale verso la macchina ed il confinamento della medesima in ambiti riservati alla ricerca scientifica.

Ora l'approccio è reso assai più semplice dall'informatica e soprattutto dai software evoluti che pilotano in retroazione i singoli dispositivi e che, in qualche caso, assumono la connotazione del sistema esperto per la gestione del complesso.

GCTS rappresenta l'attuale riferimento di quest'evoluzione.

Vediamo per sommi capi gli ausili offerti.

Una volta scelta l'ampiezza dell'oscillazione forzante, l'apparecchiatura GCTS varia automaticamente la frequenza motrice, esplorando per steps definiti il campo di variazione prefissato dall'operatore, trovando il picco di deformazione di taglio e dunque la frequenza di risonanza incognita.

Da quest'ultima, dati alcuni parametri costanti o facilmente misurabili, quali i momenti polari di massa del complesso cappello-motore (I_o) e del provino (I), l'altezza (h) e la densità (ρ) del medesimo, il sistema informatico esegue automaticamente l'analisi inversa per restituire il modulo di taglio G al livello di deformazione γ in regime di risonanza.

Per determinare il rapporto di smorzamento D si possono seguire diverse procedure: ad esempio, il parametro risulta dal decremento d'ampiezza delle oscillazioni libere che seguono allo stop del motore, oppure dal calcolo operato sulla semibanda di potenza per oscillazioni forzate alla frequenza di risonanza.

In ogni caso, per l'apparecchiatura GCTS, l'onere del pilotaggio, dell'acquisizione dati, del calcolo, della restituzione dei risultati in forma numerica e grafica è delegato al P.C.

Per descrivere più in generale le prerogative dei tests ciclici, conviene ricordare che essi esplorano comportamenti reologici lontanissimi, lontani o prossimi alle condizioni di rottura che caratterizzano le prove di laboratorio tradizionali.

Considerato che le caratteristiche meccaniche di una terra sono tutt'altro che lineari e che ad essa si richiedono le più diverse prestazioni, torna utile ampliare lo spettro speculativo, soprattutto per quanto concerne la rigidezza.

Nel test di colonna risonante, ad esempio, giacché la prova non è distruttiva, si può ripetere la procedura sul medesimo campione per ottenere diversi livelli di deformazione γ e, di conseguenza, diversi G e D.

Si possono poi combinare prove di colonna risonante e tagli torsionali ciclici, magari utilizzando provini cavi, così da descrivere le leggi di variazione di G e D in un range di deformazioni ampio diversi ordini di grandezza (vidi figura 3).

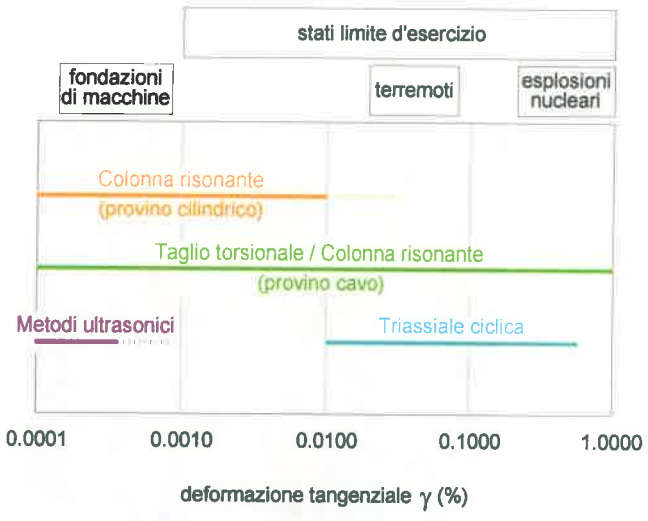

fig. 3

L'analisi della risposta di altri parametri (la pressione interstiziale u, ad esempio), indica ulteriori dati d'interesse, quali il modello reologico di riferimento, come evidenzia il diagramma in figura 4).

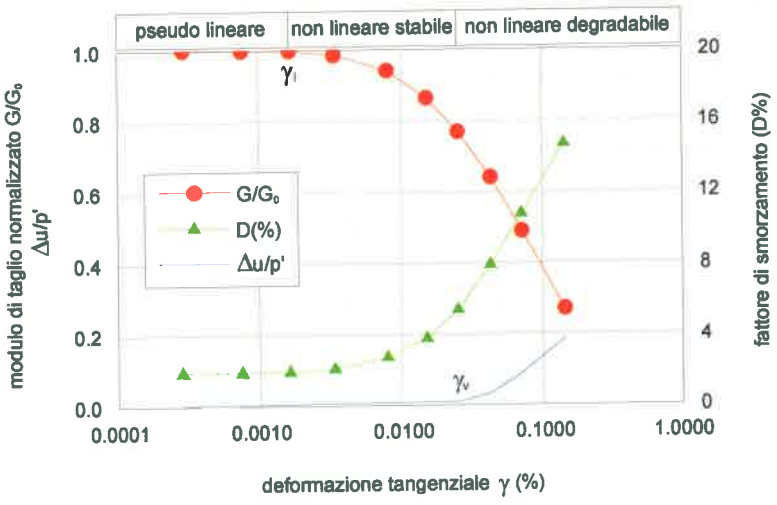

fig. 4

Abbinando la cella ad una pressa del tipo STX-50 (vedi figura 5), si apre la via alla consolidazione anisotropa.

fig. 5

Ovviamente tale pressa può effettuare tests triassiali monotonici e ciclici a grandi deformazioni.

Introducendo opportune apparecchiature, si ottengono prestazioni ed automatismi di rilievo: dalla saturazione alla consolidazione, fino ai vari stages di carico per eseguire prove triassiali complete senza l'assistenza continuativa del tecnico, sgravandolo dalle complessità di corollario.

Gl'investimenti economici sono ovviamente adeguati alle intrinseche raffinatezze, ma non tali da disarmare il laboratorio di ricerca e nemmeno quello di prova che persegue l'eccellenza tecnologica e la diversificazione del servizio.

Si tratta di incentivare la richiesta di prove sperimentali significative laddove la geotecnica gioca un ruolo fondamentale, e noi crediamo che sia giunto il tempo di provarci, sulla scia di quanto già sta accadendo per i conglomerati bituminosi.

Da qui la partnership tra noi e l'americana GCTS, azienda proiettata sul testing avanzato di terre, rocce e, appunto, conglomerati bituminosi.

Il motto "Accuracy is the underlying strategy" adottato da GCTS ci trova pienamente d'accordo, giacché Tecnotest persegue il medesimo obiettivo da quarant'anni.

GCTS Resonant Column Device*

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Introduction

Dynamic soil properties are important parameters for the analysis and design of structures required to resist dynamic loads such as earthquake shaking, machinery vibration, traffic loading, etc. Each one of the above events subjects the soil-structure system to very different amplitudes and frequencies and requires the soil dynamic properties for a wide range of load amplitudes and frequencies.

The mechanical behavior of soils is determined by effective stress, void ratio, water content, and several other factors such as strain level and stress or strain path. All these factors are equally important under either static or dynamic loading conditions and therefore are not normally considered as the elements characterizing the dynamic behavior as differentiated from the static one. Then, the features that distinguish the dynamic from the static problems are the speed of loading and the load repetition.

The speed of loading or loading period typically considered soil-structure design ranges from 0.1 to 3.0 seconds for seismic loading and for 10 to 100 repetitions. The soils in the subgrade underneath railways and road pavements are usually loaded at a speed from about 0.1 to 1.0 seconds but with a very large number of repetitions. Pile driving and machinery foundation can apply loads at very high speeds from 0.1 to 0.01 seconds. Blasting can apply shock or impulse loads at speeds of up to 0.001 seconds.

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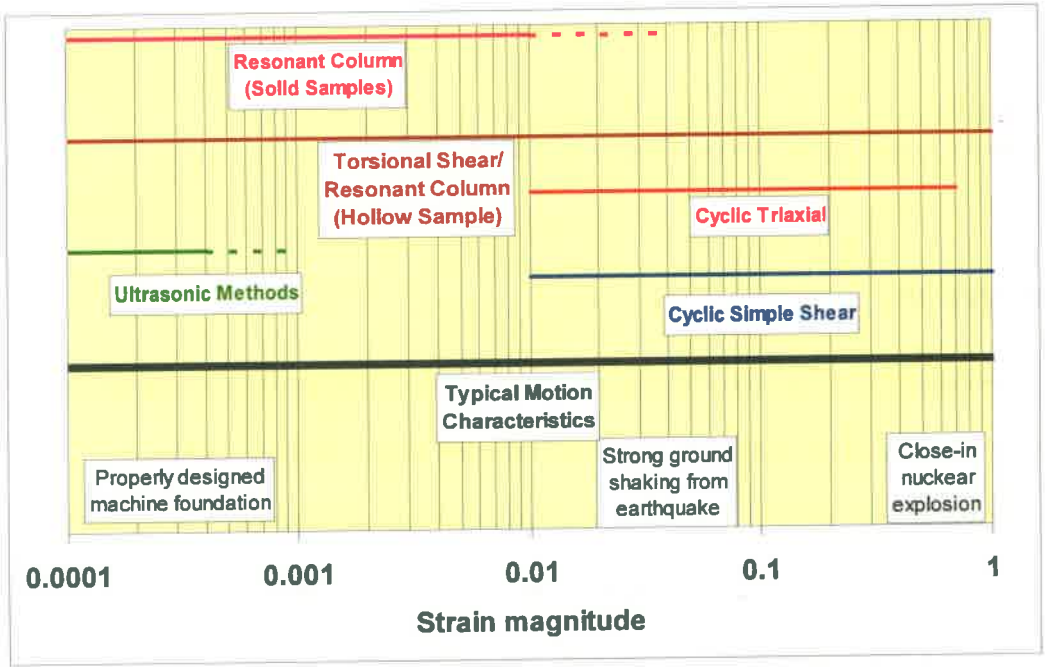


Figure 1. Typical strain level associated with different laboratory tests and field events.

Many techniques have been used in the laboratory to investigate dynamic soil properties. Some of the most common are the dynamic triaxial test, the simple shear test, the resonant column test, the torsional shear test and the ultrasonic velocity test. Each one has been designed to match as close as possible the soil boundary conditions in the field as well as the stress paths, load amplitudes, and load frequencies of different type of events. Figure 1 shows the typical or accurate[†] range of most common dynamic devices with respect to strain amplitude.

Field testing, while measuring properties of the soil in its natural state, are generally limited to measurements of dynamic moduli at very small strains. In addition,

[†] It is always possible to extend the accurate range of the strain amplitudes for most devices through the use of specialized sensors and techniques.

no readily usable field method is available for obtaining material damping at any strain amplitude.

Resonant Column Method

The resonant column test is the most commonly used laboratory test for measuring the dynamic soil properties at low to medium strains. The test is performed by vibrating a solid or hollow soil column in one of its natural modes. The wave propagation velocity is then determined from the resonant frequency.

Resonant column devices have been used to investigate soil and rock dynamic behavior as early as the 1930's. Many different implementations of the resonant column device have been designed by researchers throughout the world. Until recently, resonant column test systems required a high level of expertise by the user in order to operate very complicated electronic devices to perform this test. The modern resonant column apparatus has evolved into a simple to use testing system through the use of modern sensors and computerized electronics that perform this test automatically. Of course, as with any other test equipment, adequate specimen setup techniques and proper selection of sensor ranges are key to obtain valid results.

In the GCTS Resonant Column apparatus, a harmonic torsional excitation is applied to the top of the specimen by an electromagnetic loading system or motor. A torsional harmonic load with constant amplitude is applied over a range of frequencies and the response curve (strain amplitude) is measured. The shear wave velocity is obtained by measuring the first-mode resonant frequency. The shear modulus is calculated from this shear wave velocity and the soil density. Material damping can also

be obtained from either the free-vibration decay after the forced vibration is removed or from the width of the frequency response curve assuming viscous damping. The torsional harmonic load amplitude is increased with each test to obtain the shear modulus and damping values for different strain ranges.

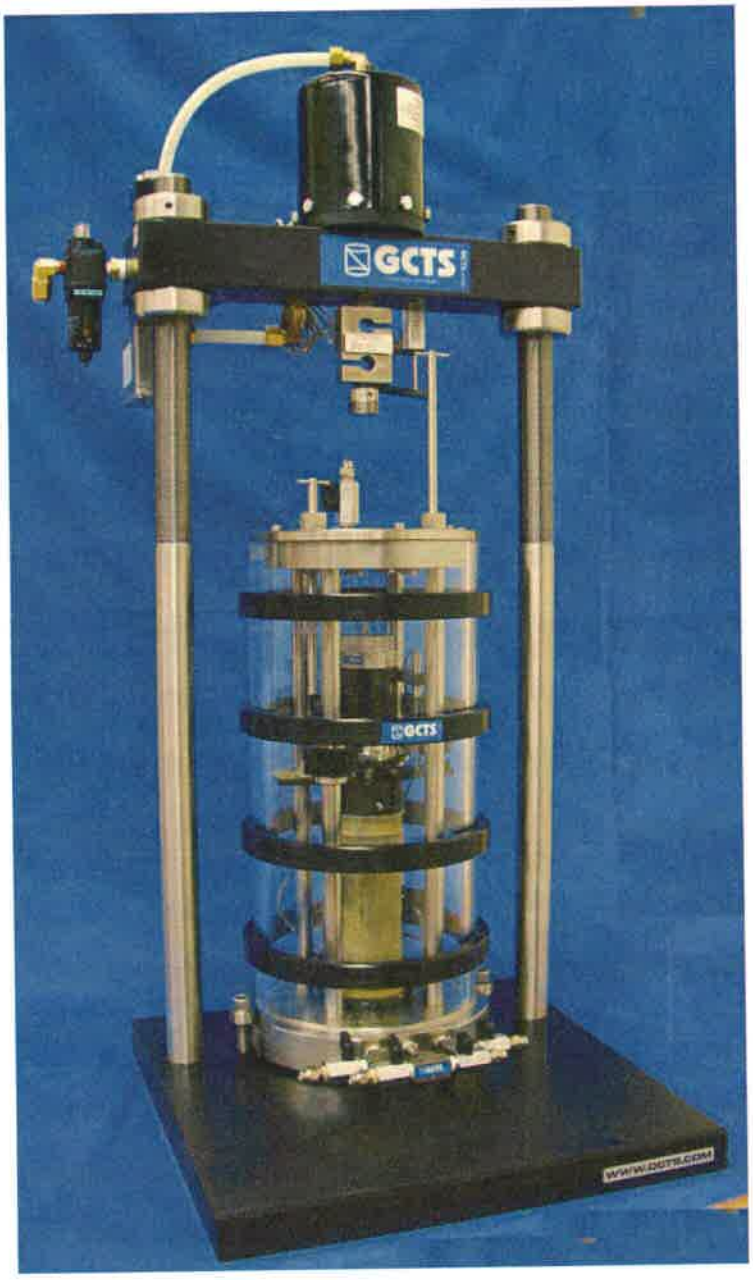


Figure 2. GCTS Resonant Column Cell

The GCTS Resonant Column device introduces several advantages over previous designs. Having the driving system completely automated not only simplifies its operation but also reduces the total numbers of cycles applied at each frequency while finding the peak response and thus avoiding early specimen degradation. The GCTS Resonant Column system also introduces a floating drive system that allows for unlimited specimen consolidation. The top of the specimen is constrained in the horizontal direction preventing any bending of the soil column during testing and ensuring a higher rate of success.

Theoretical Background

The resonant column method is based on the one-dimensional wave equation derived from the theory of linear-elastic vibration as the solution for non-linear vibrations is extremely complex. In fact, this is one of the factors that limit the resonant column test to medium and low strain amplitudes even when that apparatus is capable of measuring larger strains.

The GCTS Resonant Column device is fixed-free system where the soil column is fixed at the base and free to rotate at the top. With this apparatus, a soil specimen is first consolidated and then an external cyclic torsional load is applied to the top of the specimen. The loading frequency is gradually changed until the maximum response (strain amplitude) is found. The lowest frequency at which the strain amplitude is maximized is the fundamental frequency of the soil specimen and the driving system. The fundamental frequency is then a function of the soil stiffness, the specimen

geometry, and the characteristics of the resonant column device. Material damping is determined by both, the free vibration decay method and the half-power bandwidth method.

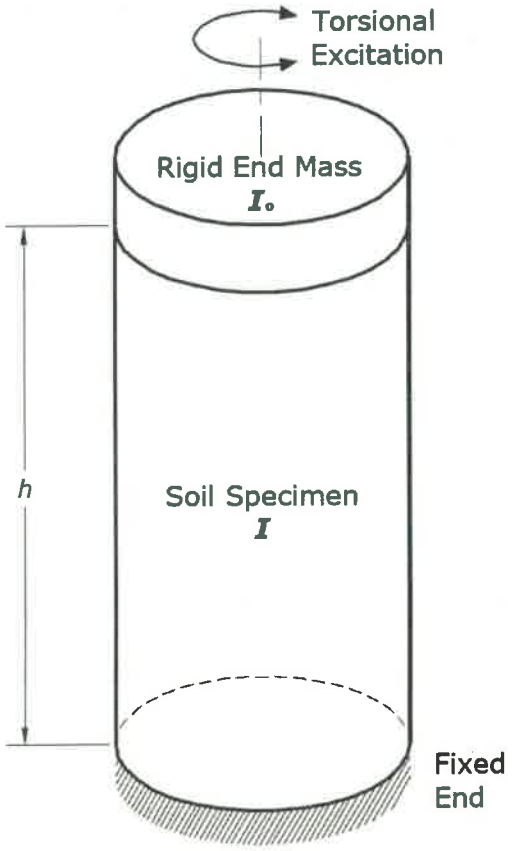


Figure 3. Idealized fixed-free resonant column specimen

Shear Modulus

The governing equation of motion for the fixed-free resonant column test as idealized in figure 3 for torsional vibration with a Kelvin-Voigt soil model is derived as follows. First, a torque T applied to an elastic soil cylinder an incremental angle of twist, $d\theta$, along an incremental length of the specimen, dz, generates a torque, T, equal to:

$$T = GJ\frac{d\theta}{dz} \tag{1}$$

where T = torque,

G = shear modulus of the soil, and

J = polar moment of inertia of the cross sectional area.

From the diagram shown in figure 4, the torque on the two faces of the soil element are T and $T+\frac{\partial T}{\partial z}dz$. Using the torque T from equation (1) we obtain:

$$\frac{\partial T}{\partial z}dz = GJ\frac{\partial^2\theta}{\partial z^2}dz \tag{2}$$

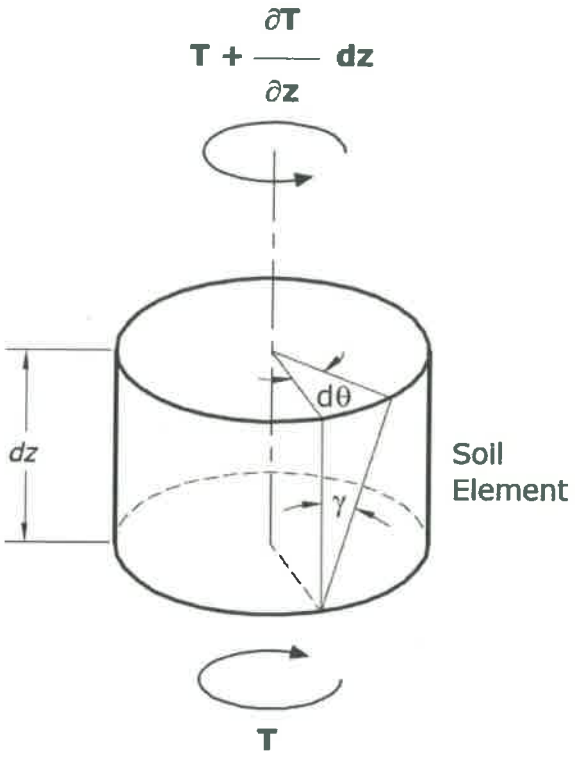


Figure 4. Differential soil element

Applying Newton's second law to the motion of the soil column and equating this net torque to the product of the mass polar moment of inertia and the angular acceleration:

$$\frac{\partial T}{\partial z} dz = I \frac{\partial^2 \theta}{\partial t^2} = \rho J \, dz \frac{\partial^2 \theta}{\partial t^2} \tag{3}$$

where I = mass moment of inertia = ρ J dz, and

ρ = soil mass density,

Substituting $\frac{\partial T}{\partial z}$ from equation 1 and using the relationship between the shear wave velocity, Vs, shear modulus, and mass density ($G = \rho \, Vs^2$) we obtain the wave equation in torsion for an elastic rod:

$$\frac{\partial^2 \theta}{\partial z^2} = \frac{1}{Vs^2} \frac{\partial^2 \theta}{\partial t^2} \tag{4}$$

The general solution to equation (4) is found using separation of variables as:

$$\theta(z,t) = \left[A \sin\left(\frac{\omega}{Vs} z\right) + B \cos\left(\frac{\omega}{Vs} z\right) \right] * e^{i\omega t} \tag{5}$$

where ω = the natural circular frequency and

A and B = constants that depend on the boundary conditions of the soil column.

The boundary conditions in the GCTS Resonant Column system are:

1. The angular displacement at the bottom (fixed end) is zero
2. The torque at the top of soil specimen (free end) is equal to the inertia torque of the drive system but opposite.

From the first boundary condition we find the B = 0 by substituting θ = 0 at z = 0. The second derivative of the general solution with respect to time is:

$$\frac{\partial^2 \theta}{\partial t^2} = \frac{\partial^2 \left[A \sin(\omega z / Vs) e^{i\omega t} \right]}{\partial t^2} = -\omega^2 \, A \sin\left(\frac{\omega z}{Vs}\right) e^{i\omega t} \tag{6}$$

From the second boundary condition, the torque at the free end of the soil specimen is:

$$T_{z=h} = -I_0\left(\frac{d^2\theta}{dt^2}\right) \tag{7}$$

where I_0 = mass moment of inertial of drive system, and

h = height of soil specimen

Substituting $\left(\frac{d^2\theta}{dt^2}\right)$ from equation (6) into equation (7):

$$T_{z=h} = I_0\,\omega^2\,A\sin\left(\frac{\omega h}{Vs}\right)e^{i\omega t} \tag{8}$$

Combining equations (1) and (8) we obtain:

$$GJ\frac{\partial\theta}{\partial z} = I_0\,\omega^2\,A\sin\left(\frac{\omega h}{Vs}\right)e^{i\omega t} \quad @ \quad z = h \tag{9}$$

Finding the derivative of θ with respect to z for z = h in equation (5) results in:

$$\left(\frac{\partial\theta}{\partial z}\right)_{z=h} = \frac{A\omega}{Vs}\cos\left(\frac{\omega h}{Vs}\right)e^{i\omega t} \tag{10}$$

Substituting equation (10)into equation (9):

$$GJ\frac{\omega}{Vs}\cos\left(\frac{\omega z}{Vs}\right) = I_0\,\omega^2\sin\left(\frac{\omega h}{Vs}\right) \tag{11}$$

Using again the relationship $G = \rho\,Vs^2$ in equation (11) it becomes:

$$\rho Vs\,J\,\omega\cos\left(\frac{\omega h}{Vs}\right) = I_0\,\omega^2\sin\left(\frac{\omega h}{Vs}\right) \tag{12}$$

Equation (12) is further reduced using the relationship $I = \rho\,J\,h$ to:

$$\frac{I}{h}\cdot Vs\cdot\omega\cdot\cos\left(\frac{\omega h}{Vs}\right) = I_0\,\omega^2\sin\left(\frac{\omega h}{Vs}\right) \tag{13}$$

Rearranging the terms in equation (13) results in the following expression:

$$\frac{I}{I_0} = \frac{\omega h}{Vs}\tan\left(\frac{\omega h}{Vs}\right) \tag{14}$$

where I = mass moment of inertia of the soil column, and

I_0 = mass moment of inertia of the drive system including the top cap.

Once the shear wave velocity, Vs, is determined, the shear modulus, G, is calculated as follows:

$$G = \rho \cdot Vs^2 \qquad (15)$$

Equations (14) and (15) are used by the GCTS software to reduce the data from the resonant column tests.

Shear Strain

The shear strain in a solid cylindrical resonant column specimen loaded in torsion varies from zero at the center line of the specimen (or from a minimum value at the inner surface of a hollow specimen) to a maximum value at its outer edge as shown in figure 5. The shear strain, γ, is calculated as follows:

$$\gamma(r) = \frac{r\,\theta_{max}}{h} \qquad (16)$$

where r = radial distance from the soil column axis,

θ_{max} = maximum angle of twist, and

h = specimen height.

Because the shear strain is not constant throughout the radial distance, an equivalent shear strain, γ is required to represent the average shear strain. This variation of the shear

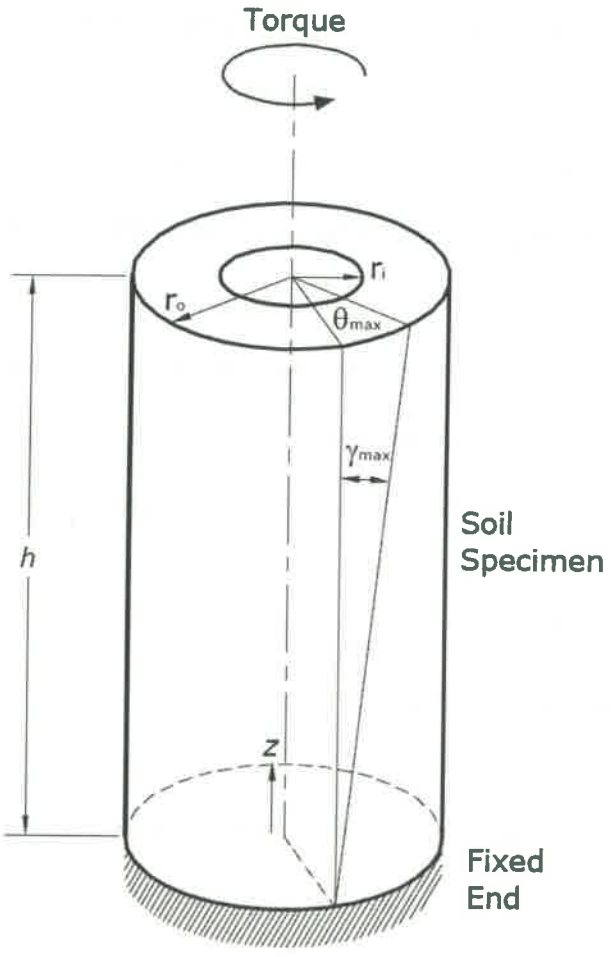


Figure 5. Shear Strain in soil specimen

strain makes it desirable to test hollow specimens instead of solid ones minimizing the variation of shear strain amplitude across the specimen.

Regardless of the type of specimen, solid or hollow, a single or unique value of shear strain amplitude associated with the measured shear modulus, G, is required. Conventionally, r_{eq} is assumed as 2/3 r_o for solid specimens with radius r_o and $(r_i + r_o)/2$ for hollow specimens with an inside radius r_i and an outside radius r_o. Chen and Stokoe

found that the value of r_{eq} varied from 0.82 r_o for a peak shear strain below 0.001% to 0.79 r_o for peak shear strain of 0.1% for solid specimens.

In the GCTS Resonant Column device, the angle of twist at the top of the specimen, θ_{max}, can be measured with either an accelerometer or proximitors mounted atop of the specimen at a radius r_{sensor}. If an accelerometer is used to measure the shear strain, the acceleration value is double integrated with respect to time to determine the torsional displacement, x, of the sensor support plate at the accelerometer location. The calculation of the torsional displacement, x, from the acceleration, $\ddot{x}$, is:

$$x = -\frac{\ddot{x}}{\omega^2} = -\frac{\ddot{x}}{4\pi^2 f^2} \tag{17}$$

where ω = circular frequency, and

f = is the linear frequency.

Assuming small angles, the angle of twist of the top plate is calculated by dividing the sensor displacement output by the radius to the position of the sensor, r_{sensor}.

$$\theta_{max} = \frac{x}{r_{sensor}} \tag{18}$$

and

$$\gamma = \frac{r_{eq}\,\theta_{max}}{h} \tag{19}$$

Viscous Damping

It is not easy to define true material damping but it is common practice to express the damping of real materials in terms of its equivalent viscous damping ratio. The free

vibration response for a system with a single degree of freedom with viscous damping can be expressed as:

$$0 = m\ddot{x} + c\dot{x} + kx \tag{20}$$

where $\ddot{x}$ = acceleration,

$\dot{x}$ = velocity,

x = displacement,

m = mass,

c = viscous damping coefficient, and

k = spring constant.

Considering the following relationships:

$$D = \frac{c}{c_c}, \quad c_c = 2\sqrt{km}, \text{ and } \omega_n^2 = \frac{k}{m} \tag{21}$$

where D = viscous damping ratio,

c_c = critical damping coefficient, and

ω_n = natural frequency (undamped).

From the above relationships and dividing equation 20 by the mass, m, we obtain:

$$0 = \ddot{x} + 2D\omega_n\dot{x} + \omega_n^2 x \tag{22}$$

There are three general solutions for equation (22) that depend on whether the single degree of freedom system is underdamped, critically damped, or overdamped. Free vibration of soil specimens in the resonant column test normally exhibits an underdamped behavior and the general solution to this case is:

$$x(t) = Ce^{-\omega_n Dt}\sin(\omega_d t + \varphi)\sin\left(\frac{\omega_n h}{Vs}\right) \tag{23}$$

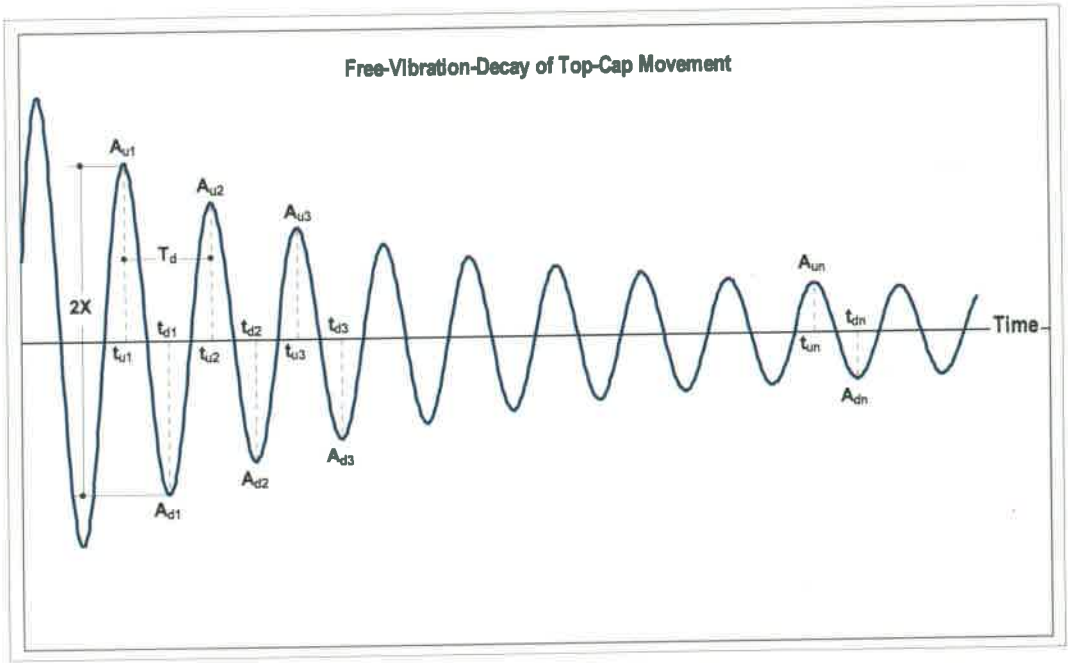


Figure 6. Free-vibration decay

where C = constant,

ω_d = damped resonant frequency, and

$$\omega_d = \omega_n \sqrt{1 - D^2} \tag{24}$$

The ratio of any two peaks depicted in figure 6 is given as:

$$\frac{x_n}{x_{n+1}} = e^{-\omega_n D(t_n - t_{n+1})} = e^{\frac{2\pi D}{\sqrt{1-D^2}}} \tag{25}$$

where $t_{n+1} = t_n + 2\pi/\omega_d$. The logarithmic decrement, δ, is found by taking the natural logarithm of equation (24).

$$\delta = \ln \frac{x_n}{x_{n+1}} = \frac{2\pi D}{\sqrt{1 - D^2}} \tag{26}$$

The damping ratio is calculated as:

$$D = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}} \tag{27}$$

The GCTS Resonant Column software records the free vibration data for all the cycles with a shear strain amplitude of at least 15% of the maximum shear strain obtained during the forced vibration test. This program calculates the natural logarithm of the normalized decay amplitude for each cycle and determines the logarithmic decrement using a linear least-square curve fitting.

Half-Power Bandwidth

A second method to measure material damping in the resonant column test is the half-power bandwidth method. From the forced-vibration test, the logarithmic decrement is calculated by measuring the width of the frequency response curve near resonance.

$$\delta = \frac{\pi\left(f_2^{\,2} - f_1^{\,2}\right)}{2f_r^{\,2}} \sqrt{\frac{x^2}{x_{\max}^{\,2} - x^2}} \frac{\sqrt{1-2D^2}}{1-D^2} \tag{28}$$

where f_1 = frequency below the resonance where the strain amplitude is A

f_2 = frequency above the resonance where the strain amplitude is A

f_r = resonant frequency, and

D = material damping.

When the damping is small and the amplitude A is $\frac{A_{\max}}{\sqrt{2}}$, equation (28) can be simplified as:

$$\delta \cong \frac{\pi\left(f_2 - f_1\right)}{f_r} \tag{29}$$

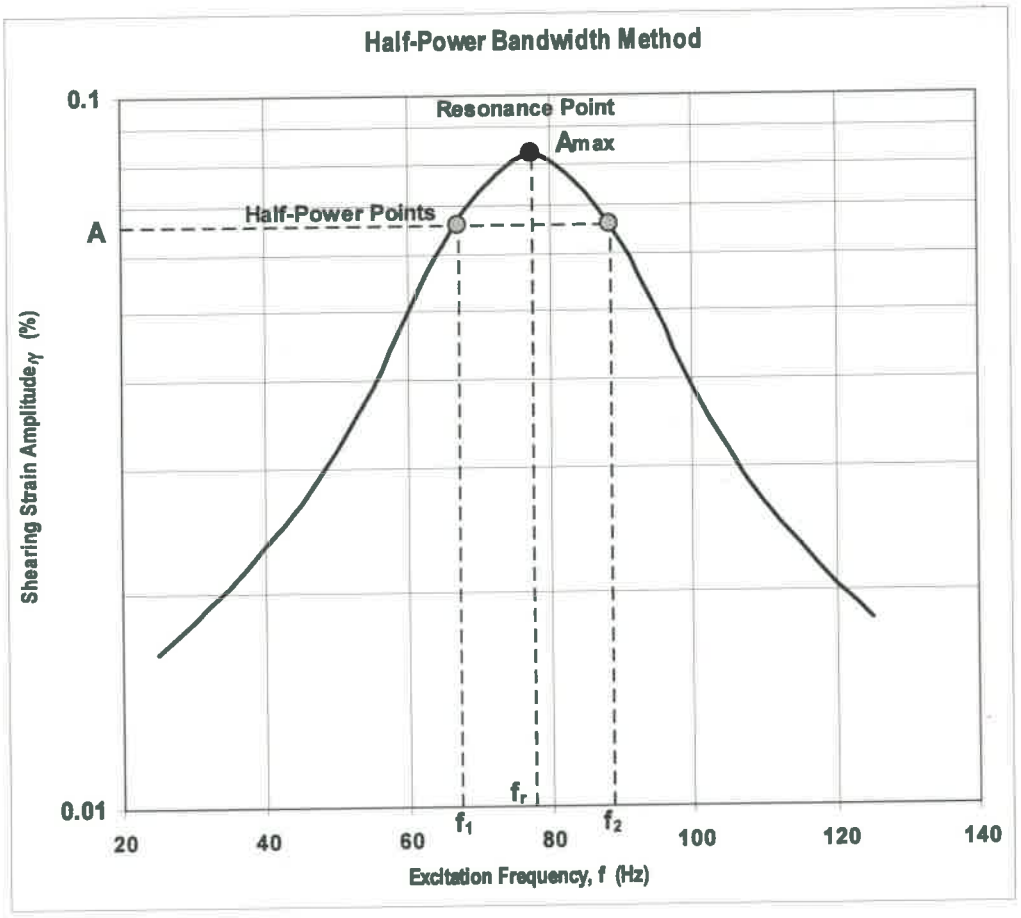


Figure 7. Material damping from Half-Power Bandwidth Method

then the damping ratio can be expressed as:

$$D \cong \frac{f_2 - f_1}{f_r} \tag{30}$$

Calibration of the Drive System

The calibration of the GCTS Resonant Column system is performed using a metallic specimen instead of a real soil specimen. The metallic specimen is assumed to

have zero, or close to zero, damping and a constant torsional stiffness, k. Then, from Newton's second law, the mass moment of inertia is related to the natural or resonant frequency, ω, as follows:

$$I = \frac{k}{\omega^2} \qquad (3\text{Ñ})$$

Even tough the torsional stiffness, k, of the calibration specimen can be found by applying a constant torque and measuring the angular rotation, this is not normally done. t ithout knowing the torsional stiffness, k, the mass moment of inertia, I, in equation (3Ñ can not be solved. The recommended procedure to find the mass moment of inertia of the drive system, I_0, is to perform two resonant column tests with the metal calibration specimen, one by itself and the other with an added mass. erform a frequency sweep with constant force amplitude to find the resonant frequency for each configuration. The force amplitude is selected to excite the calibration specimen within the limits of the installed sensors (proximitors or accelerometer) but still provide a large enough signal to measure the response accurately. Then solution to equation (3Ñ for the first calibration run without the added mass becomes:

$$I_0 + I_{cal} = \frac{k}{\omega_1^2} \qquad (32)$$

where I_0 = mass moment of inertia of the drive system and any other fixture that will be used during actual soil testing,

I_{cal} = mass moment of inertia of the calibration specimen, and

ω_1 = resonant frequency of calibration specimen without the added mass.

The second equation for the second calibration run attaching the added mass is:

$$I_0 + I_{cal} + I_{mass} = \frac{k}{\omega_2^{\,2}} \tag{33}$$

where I_{mass} = mass moment of inertia of the added mass, and

ω_2 = resonant frequency of calibration specimen with the added mass

Now, to find the mass moment of inertia of the driving system that will be used to solve equation (14) and find Vs we combine the equations (32) and (33) to get:

$$I_0 = \frac{(I_{cal} + I_{mass})\,\omega_2^{\,2} - I_{cal}\omega_1^{\,2}}{\omega_1^{\,2} - \omega_2^{\,2}} \tag{34}$$

Keep in mind that for the GCTS Resonant Column system, the specimen top cap is not used during the calibration procedure. Therefore its mass moment of inertia has to be added to the result of equation (34) to calculate the actual I_0 value that is entered into the GCTS software. The mass moment of inertial of the top cap is calculated from its geometry and mass using the appropriate formula listed in appendix A.

Example Calibration of GCTS RC Drive System

To calibrate the GCTS Resonant Column system it is first necessary to calculate the mass moment of inertia of the calibration specimen, I_{cal}, and added mass, I_{mass}. These values are calculated from the geometry and mass of each part. The calibration specimen and added mass geometries are shown in figures 8 and 9 respectively.

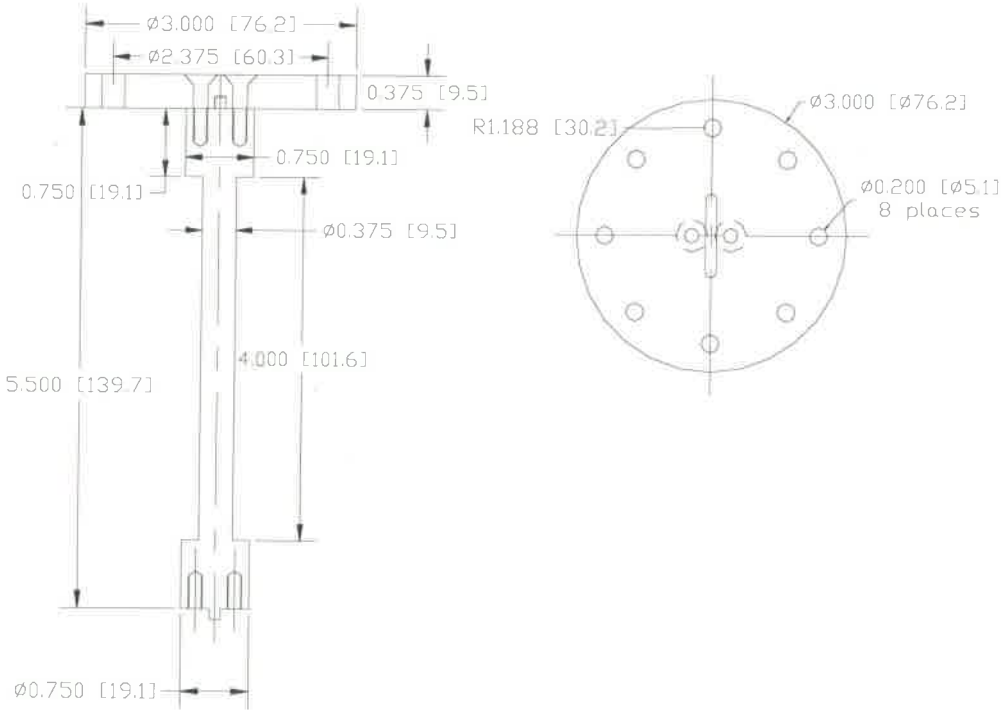


Figure 8. Calibration specimen geometry

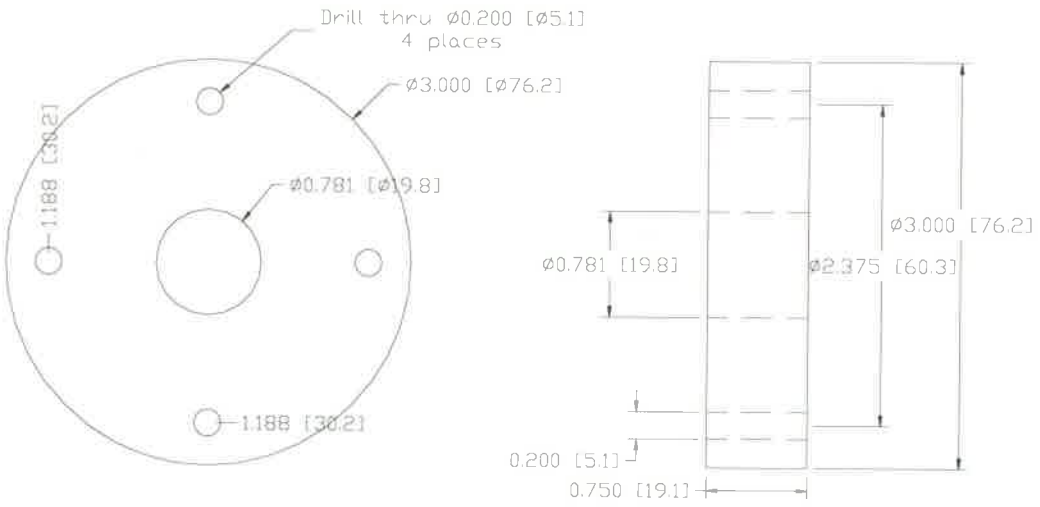


Figure 9. Added mass geometry

First, the moment of inertia of calibration specimen is calculated. The calibration specimen is made of 6061-T6 aluminum with a mass density of 2.7 g/cm^3. This calculation is done in three parts using the principle of superposition. Then I_{cal} is calculated as:

$$I_{cal} = I_{cal-plate} + I_{cal-rod-end} + I_{cal-rod} - I_{cal-holes} \tag{35}$$

and

$$I_{cal-plate} = \tfrac{1}{2} mR^2 = \tfrac{1}{2} \times 0.117\ kg \times (38.1\ mm)^2 = 84.9\ kg \cdot mm^2$$

$$I_{cal-rod-end} = \tfrac{1}{2} mR^2 = \tfrac{1}{2} \times 0.015\ kg \times (9.5\ mm)^2 = 0.7\ kg \cdot mm^2$$

$$I_{cal-rod} = \tfrac{1}{2} mR^2 = \tfrac{1}{2} \times 0.019\ kg \times (4.7\ mm)^2 = 0.2\ kg \cdot mm^2$$

$$I_{cal-holes} = 8 \cdot \left[I_{hole} + md^2 \right]$$
$$= 8 \cdot \left[\tfrac{1}{2} \times 0.001\ kg \times (2.5\ mm)^2 + 0.001 kg \times (30.2\ mm)^2 \right] = 3.8\ kg \cdot mm^2$$

then

$$I_{cal} = 84.9 + 0.7 + 0.2 - 3.8 = 82.0\ kg \cdot mm^2$$

Note that the threaded holes used to attach the top plate to the bar of the calibration specimen are included in the calculation. The voids will be filled with the screws and even though they have a larger density than the aluminum, the error is negligible.

The added mass is made of 303 stainless steel with a mass density of 7.7 g/cm^3. Then the moment of inertia of the added mass is calculated as:

$$I_{mass} = I_{mass-plate} - I_{mass-holes} \tag{36}$$

and

$$I_{mass-plate} = \tfrac{1}{2} m\left[R_i^2 + R_o^2\right] = \tfrac{1}{2} \times 0.624\ kg \times \left[(9.9)^2 + (38.1)^2\right] = 483.\ kg \cdot mm^2$$

$$I_{mass-holes} = 4 \cdot \left[I_{hole} + md^2\right]$$
$$= 4 \cdot \left[\tfrac{1}{2} \times 0.003\ kg \times (2.5\ mm)^2 + 0.003\,kg \times (30.2\ mm)^2\right] = 11.0\ kg \cdot mm^2$$

then

$$I_{mass} = 483.5 - 11.0 = 472.5\ kg \cdot mm^2$$

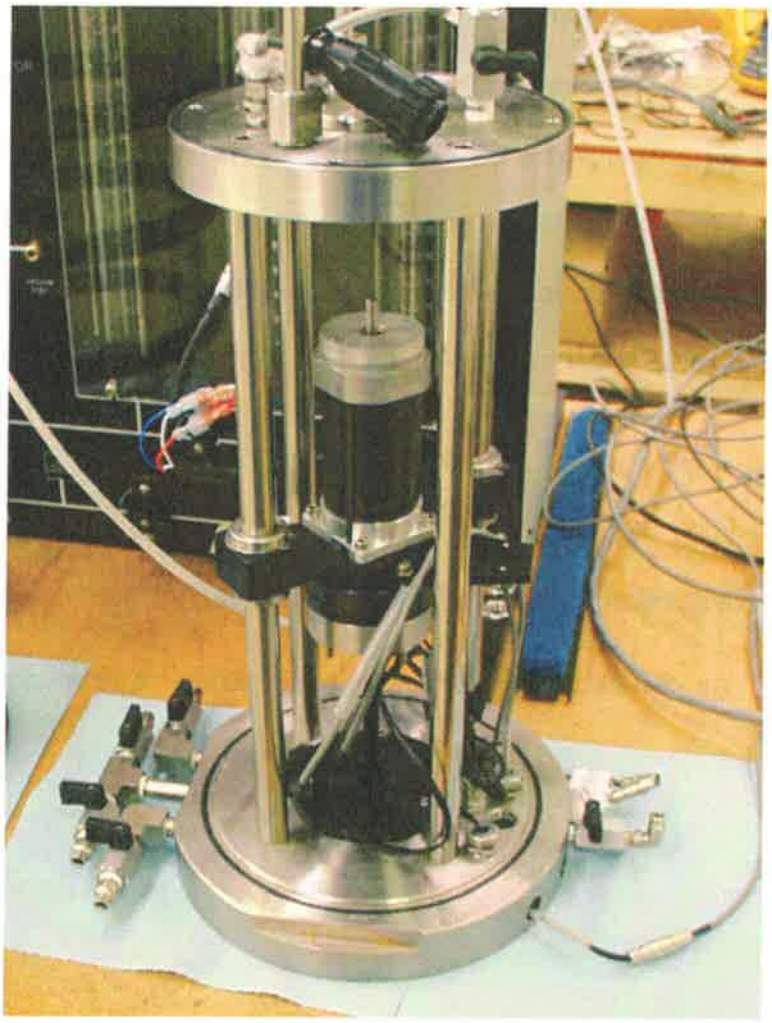


Figure 10. Calibration specimen with added mass

Now, by performing resonant column tests on the calibration specimen, first without the added mass and then with added mass we obtain the following resonant frequencies:

$\omega_{no\ added\ mass} = \omega_1 = 79.6\,Hz$, and $\omega_{with\ added\ mass} = \omega 2 = 63.4\,Hz$

then from equation (34) we obtain:

$$I_0 = \frac{(82.0 + 472.5)\times(63.4)^2 - (82.0)\times(79.6)^2}{(79.6)^2 - (63.4)^2} = 737.8\ kg \cdot mm^2$$

Because the top specimen cap was not used during this calibration procedure, the mass moment of inertia of the top cap needs to be added to the above value. GCTS offers several top specimen caps and therefore it is important to use the correct dimension of the top cap that will be used during an actual resonant column soil test. It is also important to use the same sensor or device mounted on top of the calibration specimen during actual soil tests.

Example Resonant Column Test on GCTS System

An example test is presented including pictures of the specimen preparation and test results.

Figure 11-A. Solid specimen setup

Figure 11-B. Hollow specimen setup

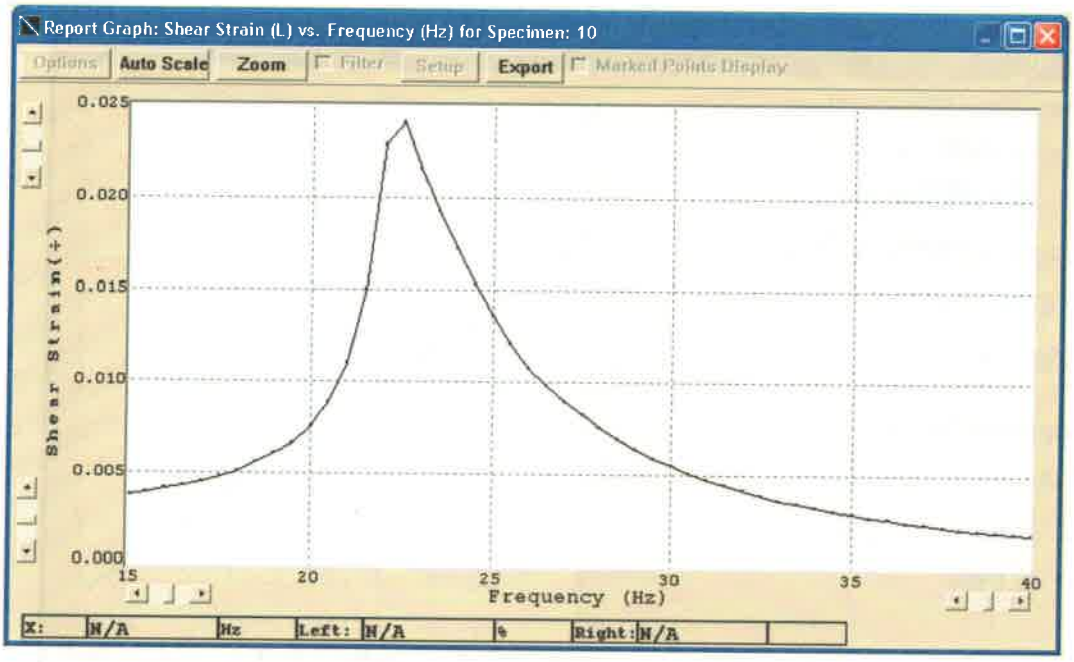


Figure 12. Frequency sweep

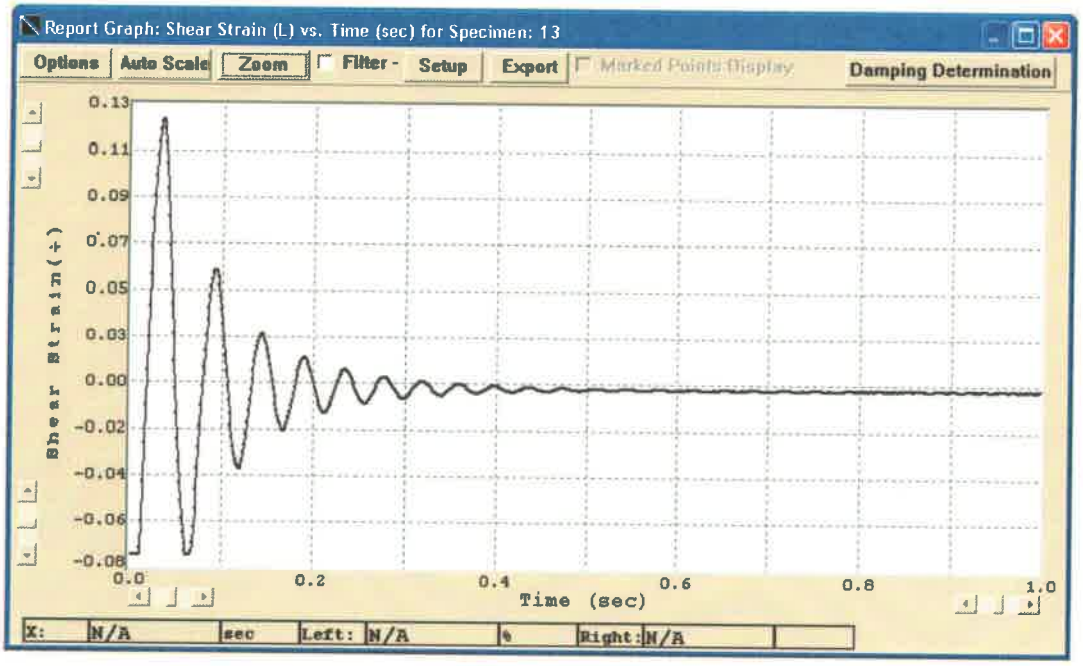


Figure 13. Free vibration

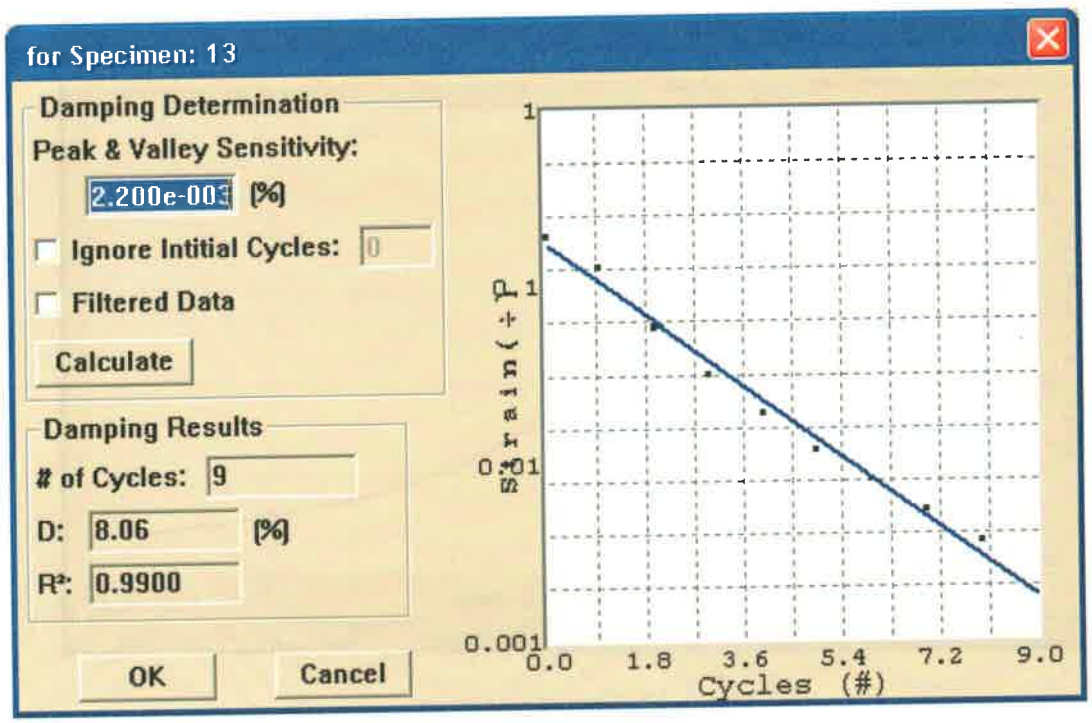
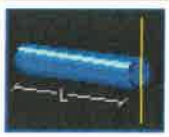


Figure 14. Free vibration damping determination

Appendix A. Formulas for

The moment of inertia of a solid body with density ρ(r) with respect to a given axis is defined by the volume integral

$$I \equiv \int \rho(\mathbf{r}) r_{\perp}^2 \, dV,$$

slender rod:	axis through center	$I=\frac{1}{12}\cdot M\cdot L^2$		axis through end	$I=\frac{1}{3}\cdot M\cdot L^2$	
rectangular plane:	axis through center	$I=\frac{1}{12}\cdot M\cdot\left(a^2+b^2\right)$		axis along edge	$I=\frac{1}{3}\cdot M\cdot a^2$	
sphere	thin-walled hollow	$I=\frac{2}{3}\cdot M\cdot R^2$		solid	$I=\frac{2}{5}\cdot M\cdot R^2$	
cylinder	hollow	$I=\frac{1}{2}\cdot M\cdot\left(R_i^2+R_o^2\right)$		solid	$I=\frac{1}{2}\cdot M\cdot R^2$	
	thin-walled hollow	$I=M\cdot R^2$				

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